8.1–8.2 Integration by parts

Product rule:

$$(uv)' = vu' + uv' \quad \text{OR} \quad (uv)' = v\,du + u\,dv$$

$\Rightarrow$ integrate both sides

$$uv = \int v\,du + \int u\,dv$$

$$\longrightarrow \boxed{\int u\,dv = uv - \int v\,du} \quad \text{by parts integration}$$

set u = part to differentiate $\to du$

dv = part to integrate $\longrightarrow v$

Eg: Evaluate

$$\int x\cos x\,dx$$

$u = x \longrightarrow du = 1dx \quad \left(\frac{du}{dx} = 1 \to du = 1dx\right)$

$dv = \cos x\,dx \longrightarrow v = \int \cos x\,dx = \sin x$ (No need to add C here!)

$$uv - \int v\,du = x(\sin x) - \int \sin x\,(1dx)$$

$$\int x\cos x\,dx = x\sin x - [-\cos x] + C = x\sin x + \cos x + C$$

Eg: $\int_1^2 \ln x \, dx$ Evaluate.

Consider: $\int \ln x \, dx = \int 1 \ln x \, dx = \int \ln x \, (1 dx)$

$u = \ln x \longrightarrow du = \frac{1}{x} dx$

$dv = 1 dx \longrightarrow v = \int 1 dx = x$

$\int \ln x dx = uv - \int v du = x \ln x - \int \frac{1}{x} x dx$

$= x \ln x - x$

So, $\int_1^2 \ln x dx = \Big[x \ln x - x \Big]_1^2 = \Big[2 \ln 2 - 2 - 0 + 1 \Big] = \boxed{2 \ln(2) - 1}$

Ex: $\displaystyle\int \frac{x^3 e^{x^2}}{(x^2+1)^2}\,dx = \int \frac{x^2 e^{x^2}\cdot x}{(x^2+1)^2}\,dx$

$u = x^2 e^{x^2} \longrightarrow du = 2xe^{x^2} + 2xe^{x^2}\cdot x^2$

$dv = \dfrac{x}{(x^2+1)^2}\,dx \longrightarrow v = \displaystyle\int \frac{x}{(x^2+1)^2}\,dx = \frac{1}{2}\int \frac{dw}{w^2}$ where $w = x^2+1$

$v = \dfrac{-1}{2(x^2+1)}$

$$uv - \int v\,du = \frac{-x^2 e^{x^2}}{2(x^2+1)} + \int \frac{1}{2(x^2+1)}\Big(\underbrace{2xe^{x^2}\left[1+x^2\right]}_{du}\Big)\,dx$$

$$= \frac{-x^2 e^{x^2}}{2(x^2+1)} + \int xe^{x^2}\,dx$$

$$= \frac{-x^2 e^{x^2}}{2(x^2+1)} + e^{x^2} + C$$

$$= \frac{x^2 e^{x^2} + 2}{2(x^2+1)} + C$$